2D and 3D Douglas ADI

# 2D Douglas ADI

Consider the equation

Splitting this we have

In Douglas' paper, this was written as (in equation 2.5)

Multiplying by , we have

Attempting to solve for and , we have

Or, finally

Combining

From above to the second equation, we have

We finally have

This is exactly the same as the form we used in class.

This is written in the last equation (2.7) as

# 3D Douglas ADI

Consider the equation

Splitting this operator into several steps, we have

After rearranging, we have

This is the same form of the 3D ADI in Douglas, equation 3.2. Simply multiply by

# 3D Douglas ADI - for analysis

Without loss of generality, we may replace with simply . For analysis, it is useful to write the 3D ADI equation in the form where and are eliminated. The result we wish to reach is that of eq 4.1 in the Douglas paper. Combining the three above equations, we have

Distributing

again

Multiplying the first term out (using dummy variables), we have

This means we have

Next, the second term

which yields

Next, the third term we have

The first term on the RHS, we can write as

Putting this all together, we have

Multiplying by we have

Rearranging, we have

Flipping sides (not changing signs), we have

Rearranging again we have

Multiplying some terms out we have

Therefore we have

Combining we have, finally

# Stability and Convergence

Let

represent the discretization error. Then we have

Where

Where is the truncation error and may be written as

Let

Solve for and we have

Or

Multiplying by we have

Let

And then we have

Let

It is easy to see by direct substitution into 4.8 that

Note that

And

Therefore we have

Let

And divide by to get

And solving for yields

Therefore

Multiplying the numerator and denominator by we have

Which is exactly the same as in Douglas.